MUTUAL COUPLING BETWEEN TWO CIRCULAR WAVEGUIDES
TERMINATED IN A CONDUCTING SPHERICAL CAVITY

Probir K. Bondyopadhyay and A. Hessel
Microwave Research Institute
Polytechnic Institute of New York
Long Island Center, Farmingdale, N.Y. 11735

ABSTRACT

The self and mutual admittances between two circular waveguide apertures in a conducting spherical cavity are formulated, from an expansion of the cavity field in terms of spherical waveguide modes. Two basic admittance functions are formed, one for the apertures polarized along the interconnecting great circle, (E-plane) and another crosswise to it. These mutual admittances have direct application to microwave network analysis of mutual coupling in spherical lens arrays on concave surfaces of the Dome Antenna type.

Modal analysis of the coupling coefficients in concave spherical collector lens arrays of aperture elements of the Dome antenna type[1] involves determination of mutual coupling between two circular apertures in a large conducting spherical cavity. This problem is also of interest in microwave network theory. In this paper an analytical treatment of this problem is reported.

The analysis is based on the fact that a spherical cavity may be regarded as a terminated spherical waveguide[2], with one termination at the origin and another at the conducting surface, so that one has essentially a two aperture spherical waveguide junction discontinuity problem. An admittance matrix description of this network is therefore possible in terms of the spherical waveguide and circular waveguide modal bases using the following stationary formula; with the assumed TE_{11} circular waveguide mode aperture illumination:

$$Y_{12} = \frac{1}{V_1 V_2} \iint_{\text{aperture } 2} \underset{\sim}{E}_2 \times \underset{\sim}{H}_{12} \cdot \underset{\sim}{n}\, ds \quad \ldots\ldots \qquad (1)$$

where $\underset{\sim}{E}_2$ is the tangential electric field distribution of the TE_{11} circular waveguide mode over aperture 2 and $\underset{\sim}{H}_{12}$ is the tangential magnetic field distribution over the short circuited aperture 2 due to the TE_{11} mode illumination of aperture 1, V_1 and V_2 being the respective modal voltages.

Referring to Fig. 1, TE_{11} modal field distribution at the apertures are first expressed in terms of spherical waveguide modes with polar axes through the center of the respective apertures. Then, for the magnetic field due to aperture 1, addition theorem for spherical harmonics is used to transfer the polar axis to the center of the second aperture. Orthogonality property of the spherical waveguide modes simplifies evaluation of the mutual admittance integrals and one finally obtains:

$$Y_{self} = \sum_{n=1}^{\infty} \left(\frac{|V_n'|^2}{Z_n'} + \frac{|V_n''|^2}{Z_n''} \right) \qquad (2)$$

$$Y_{12} = Y_{\ell\ell} \cos\alpha \cos\beta + Y_{cc} \sin\alpha \sin\beta \qquad (3)$$

where

$$Y_{\ell\ell}(\theta_{12}) = \sum_{n=1}^{\infty} \left(\frac{|V_n'|^2}{Z_n'} B_n(\theta_{12}) + \frac{|V_n''|^2}{Z_n''} C_n(\theta_{12}) \right), \text{(E-plane)} \qquad (4)$$

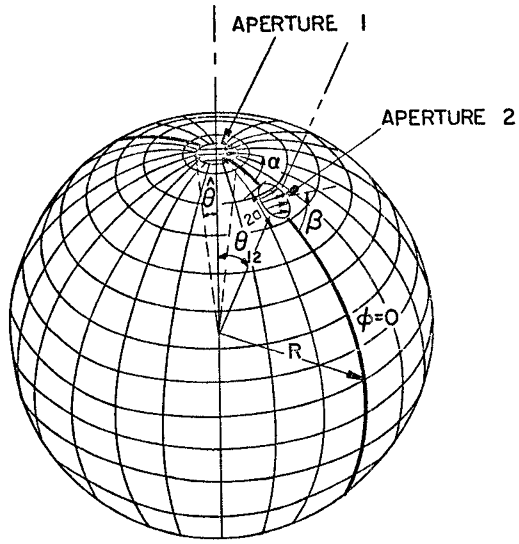

FIG. 1 GEOMETRY OF THE SPHERICAL CAVITY WITH TWO ARBITRARILY POLARIZED CIRCULAR APERTURES

and

$$Y_{cc}(\theta_{12}) = \sum_{n=1}^{\infty} \left(\frac{|V_n'|^2}{Z_n'} C_n(\theta_{12}) + \frac{|V_n''|^2}{Z_n''} B_n(\theta_{12}) \right), \text{(H-plane)} \qquad (5)$$

Spherical TM mode amplitudes, V_n', are obtained in closed form whereas, evaluation of the TE mode amplitudes V_n'' involves a single integration that can be further simplified using asymptotic expressions for associated Legendre functions $P_n^1(\cos\theta_{12})$. The terms $B_n(\theta_{12})$ and $C_n(\theta_{12})$ are the transformation coefficients for spherical harmonics of unity azimuthal index with even and odd φ-variations, respectively. These modal amplitudes and transformation coefficients were first evaluated in connection with the exterior problem[3,4]. Z_n' and Z_n'' are spherical TM and TE mode impedances respectively and are given by:

$$Z_n' = -j\zeta\left[\frac{1}{kR} + \frac{j_n'(kR)}{j_n(kR)} \right] \quad \ldots\ldots \qquad (6)$$

$$Z_n'' = \zeta^2 / Z_n' \quad , \qquad \ldots\ldots \qquad (7)$$

where ζ is the free space impedance. These modal admittances are computed very accurately using a rapidly convergent algorithm[5].

Similar expressions for self and mutual admittances involving higher order aperture modes have also been evaluated. For a single aperture spherical cavity, rigorous analysis including several higher order aperture modes shows that the single circular waveguide TE_{11} mode field approximation at the aperture yields quite accurate results for a large cavity both near and off resonance for small apertures ($a \sim .2\lambda$). As a special case, when a particular spherical TM or TE mode is near resonance, mutual admittances in (2) and (3) tend to be very large and the coupling coefficients, then, have to be evaluated in the limit. Such limiting procedure has been carried out.

Three thousand spherical waveguide modes for either TE or TM polarization are used in the numerical computation. Normalized mutual admittances as a function of aperture separation angle are plotted in Fig. 2 and 3. For large cavity, mutual admittances are very rapidly varying functions of the cavity size and aperture separation angle. Near a spherical TE mode resonance, normalized $Y_{\ell\ell}$ and Y_{cc} vary as C_n and B_n respectively with separation angle θ_{12} as seen in Fig. 3.

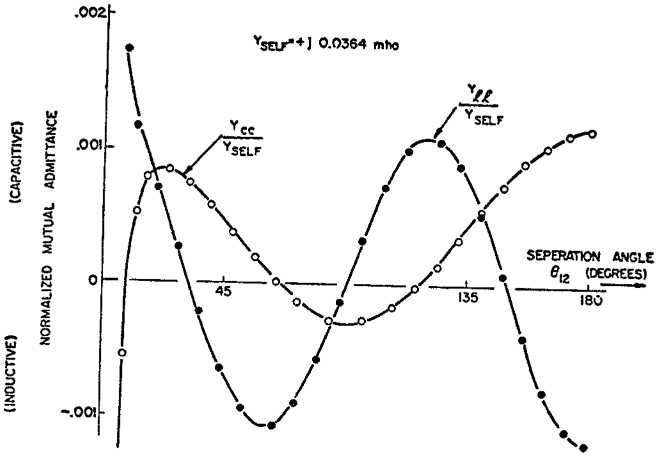

FIG. 2 NORMALIZED MUTUAL ADMITTANCE AS FUNCTIONS OF APERTURE SEPARATION ANGLE θ_{12}. ($kR = 5$, $\hat{\theta} = 1°$)

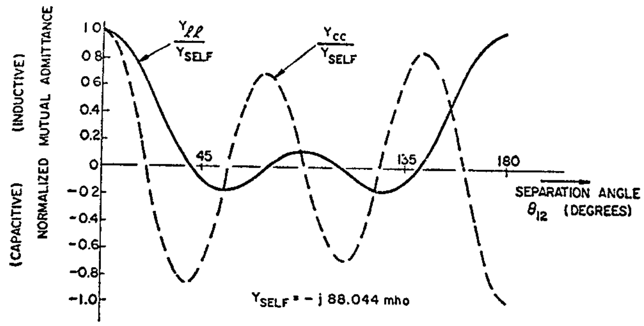

FIG. 3 NORMALIZED MUTUAL ADMITTANCES AS FUNCTION OF APERTURE ANGLE θ_{12} NEAR 5th ORDER SPHERICAL TE MODE RESONANCE. ($kR = 101.95460863$, APERTURE RADIUS = 0.1820λ)

References

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